Income Distribution and Macroeconomics

The Classical Theory

Inequality is beneficial for growth (in the post-industrialization stage)

Keynes (1920), Kaldor (1957)

- The marginal propensity to save increases with income
- Inequality channels resources towards individuals whose marginal propensity to save is higher

$\Longrightarrow$ increases aggregate savings & capital accumulation

$\Longrightarrow$ enhances the development process

The Neoclassical Paradigm

The Representative Agent Approach

- Rejects the role of heterogeneity, and thus income distribution, in economic growth
  - Growth Process $\Rightarrow$ Income Distribution
  - Income Distribution $\nRightarrow$ Growth Process

The Modern Perspective: Origins

Galor and Zeira (1988, 1993) – Heterogenous Agents Model

- Unlike the Neoclassical Paradigm

  Income Distribution $\Rightarrow$ Macroecoonmic activity process

- Unlike the Classical Perspective

  Underlined the adverse effect of Inequality on the growth process

The Galor-Zeira Model

- Overlapping-Generations economy
- $t = 0, 1, 2, 3, ...$
- One good
- 3 factors:
  - $K \equiv$ Physical capital
  - $L^s \equiv$ Skilled Labor
  - $L^u \equiv$ Unskilled Labor

Production

Total output produced

$$Y_t = Y_t^s + Y_t^u$$

- Production in the skilled-intensive sector:

$$Y_t^s = F(K_t, L_t^s) \equiv L_t^s f(k_t); \qquad k_t \equiv K_t / L_t^s$$

- Production in the unskilled-intensive sector:

$$Y_t^u = aL_t^u$$

Inverse Demand for Factors

- Capital:

$$r_t = f'(k_t) \equiv r(k_t)$$

- Skilled labor:

$$w_t^s = f(k_t) - f'(k_t)k_t \ \equiv w^s(k_t)$$

- Unskilled labor:

$$w_t^u = a \equiv w^u$$

Factor Prices

- Small open economy
- World interest $= r$

$$\Longrightarrow$$

$$\begin{aligned} r_t &= r \\ k_t &= f'^{-1}(r) \equiv k \\ w_t^s &= w^s(k) \equiv w^s \end{aligned}$$

$$\Longrightarrow$$

$$(r_t, w_t^s, w_t^u) = (r, w^s, w^u) \qquad \forall t$$

Individuals

- Continuum of measure 1
- Each Individual has 1 parent and 1 child
- Identical in:

  Preferences

  Innate abilities
- Differ in:

  Parental income $\Rightarrow$ Inv't in HC

Member of Generation t: Period of Life

- First period of life (Period t):
  - [invest in HC] or [work as unskilled]
- Second period of life (Period $t+1$):
  - [work as unskilled / no inv't in HC] or [work as skilled / inv't in HC]

Member of Generation t: Endowment and Preferences

- Time endowment:
  - 1 units of time in each period
- Capital endowment:
  - $b_t \equiv$ capital inherited in 1^{st} period
- Preferences:

$$u^t = \alpha \ln c_{t+1} + (1-\alpha) \ln b_{t+1} \qquad \alpha \in (0,1)$$

$c_{t+1} \equiv$ consumption
$b_{t+1} \equiv$ transfers to offspring

Member of Generation t: Budget Constraint

Second period budget constraint:

$$c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

$c_{t+1} \equiv$ consumption
$b_{t+1} \equiv$ transfers to offspring
$\omega_{t+1} \equiv$ wealth in period $t+1$

Member of Generation t: Optimization

$$\{c_{t+1}, b_{t+1}\} \quad = \quad \arg\max[\alpha \ln c_{t+1} + (1-\alpha) \ln b_{t+1}]$$

$$\text{s.t.} \qquad c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

Member of Generation t: Optimization

$$b_{t+1} = (1-\alpha)\omega_{t+1}$$

$$c_{t+1} = \alpha\omega_{t+1}$$

Indirect Utility: $\Longrightarrow$

$$\begin{aligned} v^t &= \alpha \ln \alpha\omega_{t+1} + (1-\alpha)\ln(1-\alpha)\omega_{t+1} \\ &= [\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)] + \ln \omega_{t+1} \end{aligned}$$

$\Longrightarrow$ v^t is monotonic increasing in 2nd period wealth, ω_{t+1}

$\Longrightarrow$ maximization of ω_{t+1}, is the basis of occupational choices

Fundamental Assumptions

- Imperfect Capital Markets:

$$r < i \tag{A1}$$

$r \equiv$ interest rate for lender

$i \equiv$ interest rate for borrowers (for inv't in HC)

- Fixed cost of education (Indivisibility of inv't in HC)

$$h > 0 \tag{A2}$$

Income: Unskilled Individuals

$$
\begin{aligned}
\omega_{t+1}^{u} &= (w^{u} + b_t)(1 + r) + w^{u} \\
&= w^{u}(2 + r) + (1 + r)b_t
\end{aligned}
$$

Income: Skilled Individuals

$$\omega_{t+1}^s = \begin{cases} w^s - (h - b_t)(1+i) & \text{if} \quad b_t \leq h \\ w^s + (b_t - h)(1+r) & \text{if} \quad b_t \geq h \end{cases}$$

$\Longrightarrow$

$$\omega_{t+1}^s = \begin{cases} w^s - (1+i)h + (1+i)b_t & \text{if} \quad b_t \leq h \\ w^s - (1+r)h + (1+r)b_t & \text{if} \quad b_t \geq h \end{cases}$$

Assumptions

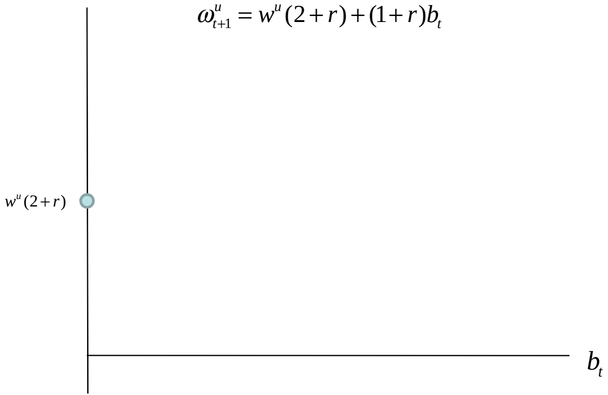
- Investment in human capital is *not* beneficial for individuals who must finance the entire cost of education via borrowing

$$w^s - (1+i)h < 0 \tag{A3}$$

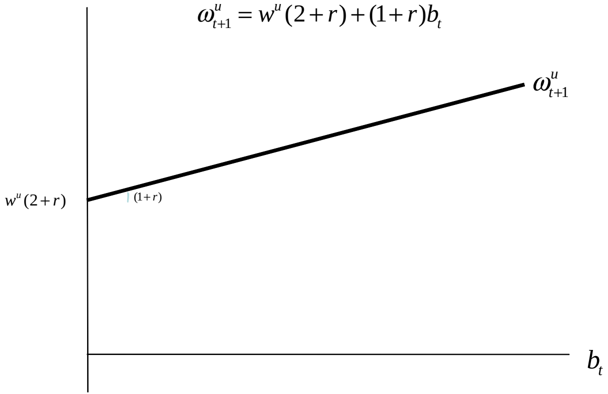
- Investment in human capital is beneficial for individuals who can finance the entire cost of education *without* borrowing

$$w^s - (1+r)h > w^u(2+r) \tag{A4}$$

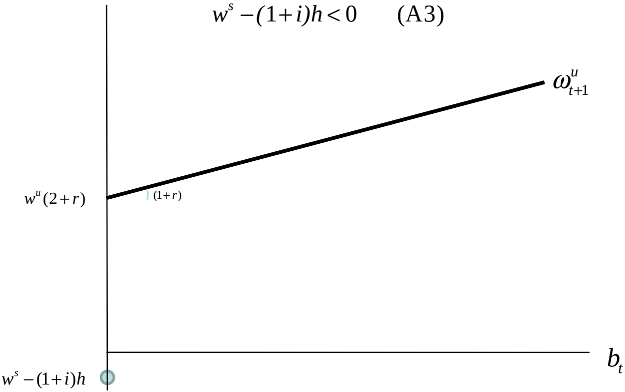
Income from Being Unskilled Worker



Income from Being Unskilled Worker



Income from Being Skilled Worker: Borrowers



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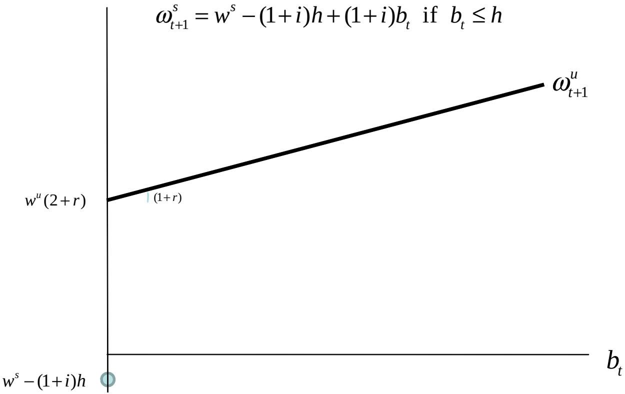

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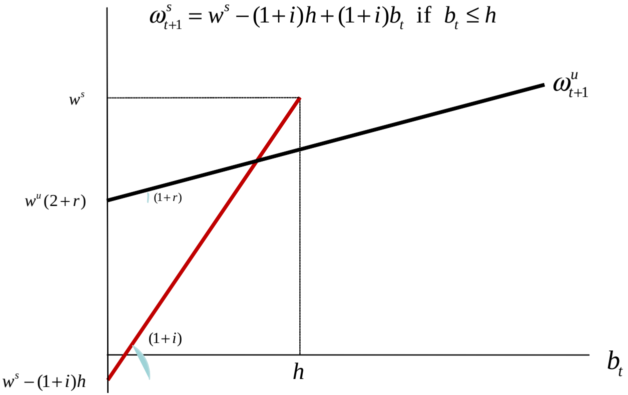
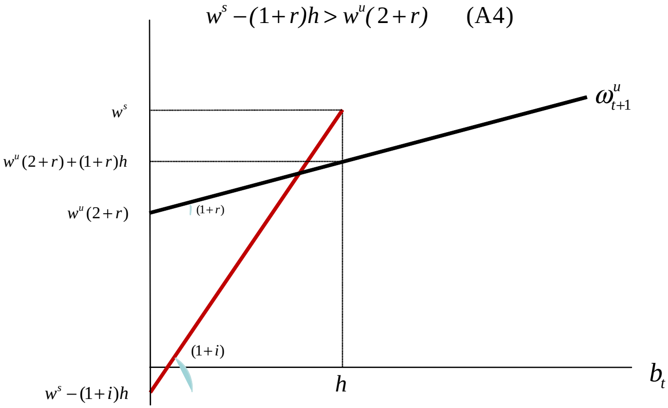
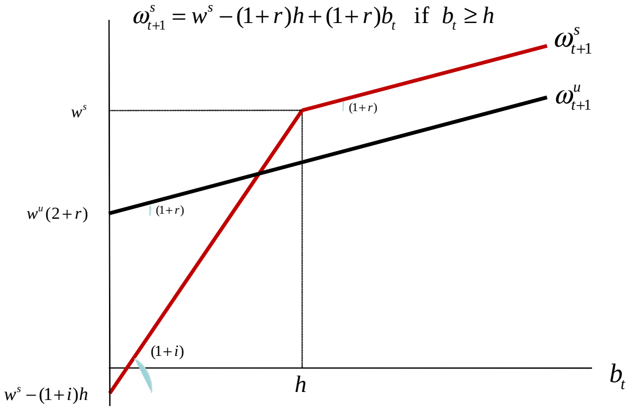


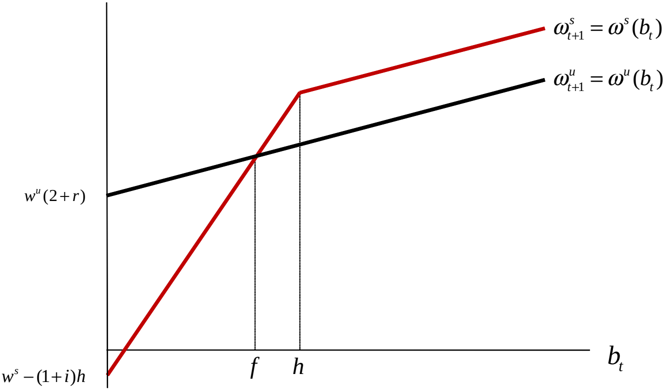
Income from Being Skilled Worker: Borrowers



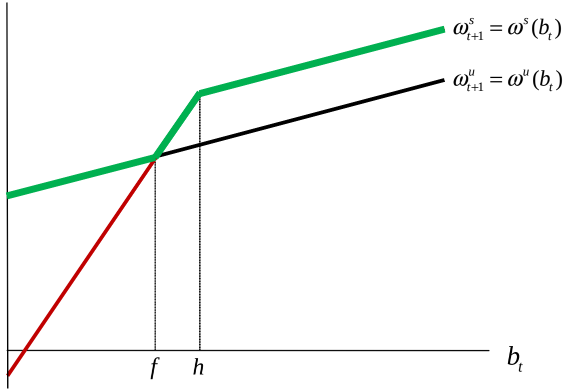
Income from Being Skilled Worker: Lenders



Bequest and Occupational Choice



Bequest and Occupational Choice



Bequest and Occupational Choice

$$b_t \quad \begin{cases} < f & \rightarrow \omega^u_{t+1} > \omega^s_{t+1} \text{ (individual } t \text{ becomes unskilled)} \\ > f & \rightarrow \omega^u_{t+1} < \omega^s_{t+1} \text{ (individual } t \text{ becomes skilled)} \end{cases}$$

where

$$f = \frac{w^u(2+r) - [w^s - (1+i)h]}{i - r} > 0$$

Bequest Dynamics

$$b_{t+1} = (1-\alpha)\omega_{t+1}$$

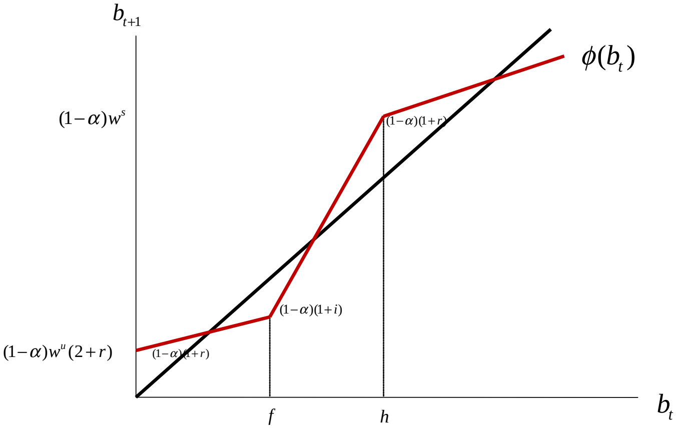
$$b_{t+1} = \begin{cases} (1-\alpha)\left[w^u(2+r) + (1+r)b_t\right] & b_t \in [0, f] \\ (1-\alpha)\left[w^s - (1+i)h + (1+i)b_t\right] & b_t \in [f, h] \\ (1-\alpha)\left[w^s - (1+r)h + (1+r)b_t\right] & b_t \in [h, \infty] \end{cases}$$

Bequest Dynamics: Sufficient Conditions for Multiplicity of Steady-Sate

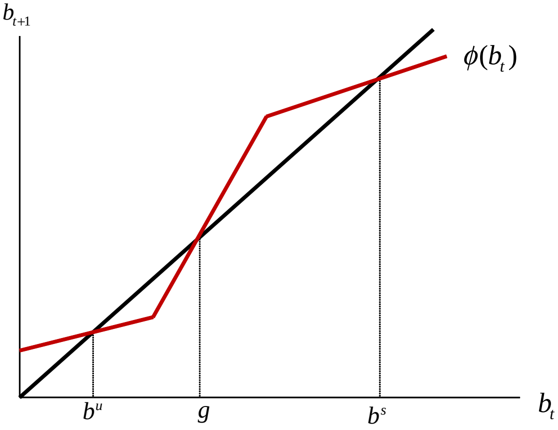
$$\begin{aligned} (1-\alpha)(1+r) &< 1 \\ (1-\alpha)(1+i) &> 1 \end{aligned} \tag{A5}$$

$$(1-\alpha)w^s > h \tag{A6}$$

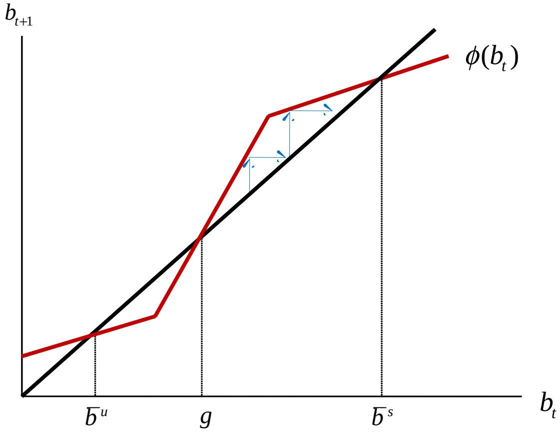
Bequest Dynamics



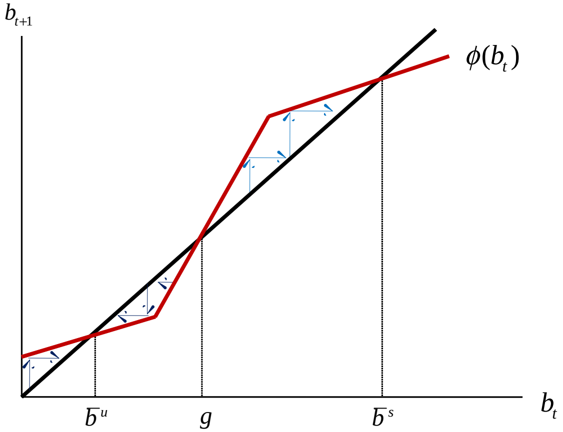
Bequest Dynamics: Multiple Steady-State Equilibrium



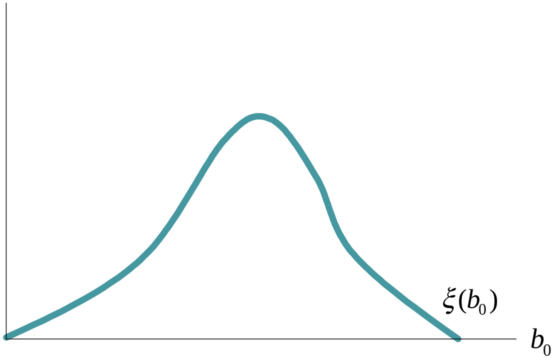
Bequest Dynamics: Stability of High Bequest Equilibrium



Bequest Dynamics: Stability of Steady- State Equilibria



The Distribution of the Inheritance in Period t



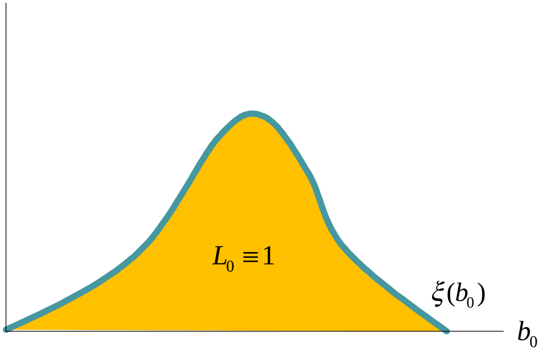
Income Distribution and the Long Run Decomposition of the Labor Force

$\xi_t(b_t) \equiv$ Distribution of inheritance at time t

$\Longrightarrow$

$$L_t = \int_0^\infty \xi(b_t) db_t \equiv 1$$

The Distribution of the Inheritance in Period t



Income Distribution of the Long Run Decomposition of the Labor Force

$$\lim_{t\to\infty} l_t^u = \int_0^g \xi_t(b_t) db_t \equiv \bar{l}^u$$

$$\lim_{t\to\infty} l_t^s = \int_g^\infty \xi_t(b_t) db_t \equiv \bar{l}^s$$

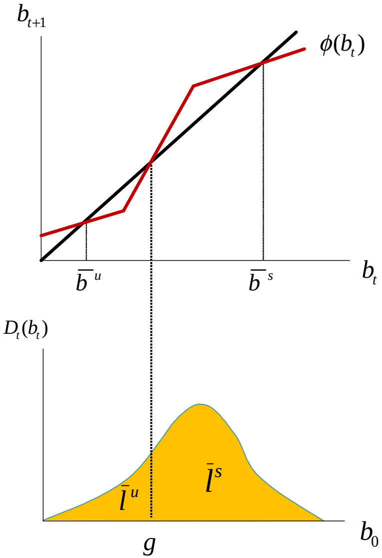
where

$$\partial \bar{l}^s / \partial g < 0$$

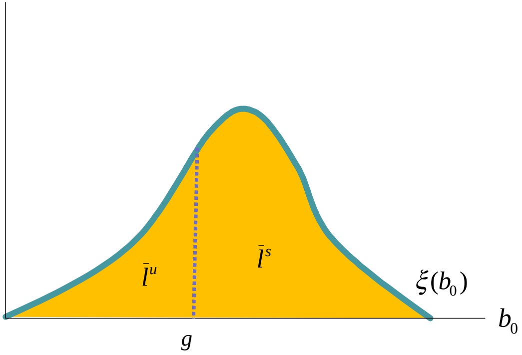
and

$$g = \frac{(1-\alpha)[(1+i)h - w^s]}{(1-\alpha)(1+i) - 1} > 0$$

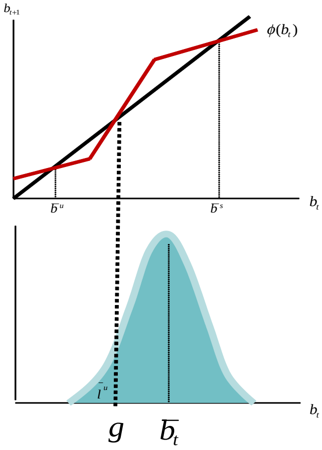
Income Distribution of Skill Composition



Income Distribution of Skill Composition

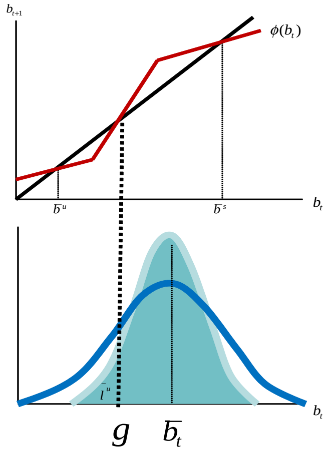


Inequality and Development: Rich Economies



Rich Economies: Inequality is Harmful for Development

Inequality reduces human capital formation



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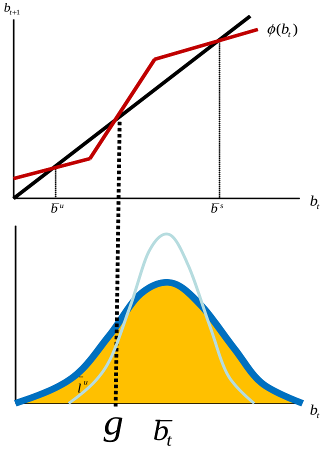

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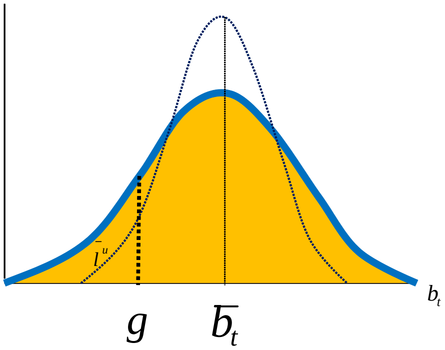
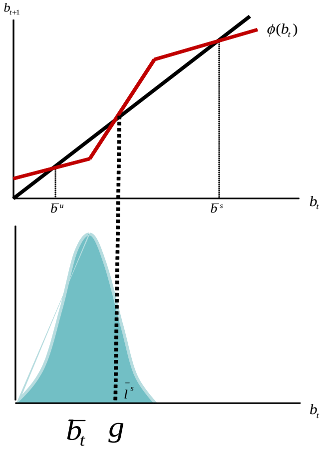
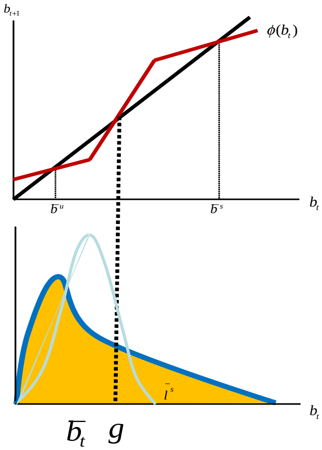


Inequality and Development: Poor Economies

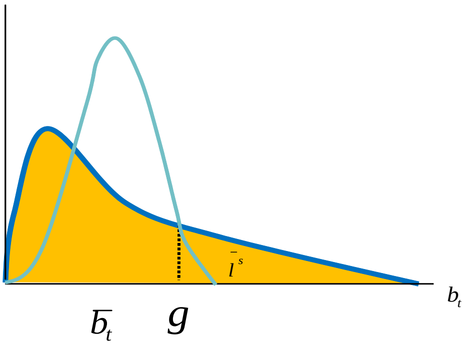


Poor Economies: Inequality may Benefit Development

Inequality stimulates human capital formation



Poor Economies: Inequality may Benefit Development



Robustness

The qualitative results are robust to:

- Education cost that is indexed to wages
- Labor augmenting technical change
- Shocks the outcome of investment in human capital, as long as wages are endogenous
- Concave production function of human capital

Robustness: Technological Progress and Endogenous Education Cost

Labor Augmenting Technological Progress: increases the productivity of workers in both the skilled-intensive and the unskilled intensive sector.

- Production in the skilled-intensive sector

$$Y_t^s = F(K_t, A_t L_t^s) \equiv A_t L_t^s f(k_t); \qquad k_t \equiv K_t / A_t L_t^s$$

- Production in the unskilled-intensive sector

$$Y_t^u = A_t a L_t^u$$

- Technological progress

$$A_{t+1} = (1+\lambda)A_t \qquad \lambda > 0.$$

Robustness: Technological Progress and Endogenous Education Cost

Factor Prices

$$w_t^s = A_t[f(k) - f'(k)k] \equiv A_t w^s$$

$$w_t^u = A_t a \equiv A_t w^u$$

$$r_t = r$$

Cost of Education

- Weighted average of the payments to teachers, administrators, and maintenance workers in the school system
- $\Rightarrow$ Weighted average of the wages skilled and unskilled workers

$$C_t^H = \theta A_t w^s + (1-\theta) A_t w^u \equiv A_t h$$

Income: Unskilled Individuals

$$
\begin{aligned}
x_{t+1}^{u} &= (A_t w^u + b_t)(1+r) + A_{t+1} w^u \\
&= A_t w^u (2 + r + \lambda) + (1+r) b_t
\end{aligned}
$$

Income: Skilled Individuals

$$x^s_{t+1} = \begin{cases} A_{t+1} w^s - (A_t h - b_t)(1+i) & \text{if} \quad b_t \leq A_t h \\ A_{t+1} w^s + (b_t - A_t h)(1+r) & \text{if} \quad b_t \geq A_t h \end{cases}$$

$\Longrightarrow$

$$x^s_{t+1} = \begin{cases} A_t [w^s(1+\lambda) - (1+i)h] + (1+i) b_t & \text{if} \quad b_t \leq A_t h \\ A_t [w^s(1+\lambda) - (1+r)h] + (1+r) b_t & \text{if} \quad b_t \geq A_t h \end{cases}$$

Threshold level of Bequest for Becoming Skilled Worker in Period t

$$f = \frac{A_t\{w^u(2+r) - [w^s - (1+i)h] - \lambda(w^s - w^u)\}}{(i-r)}$$

$$\frac{f_t}{A_t} = \frac{A_t\{w^u(2+r) - [w^s - (1+i)h] - \lambda(w^s - w^u)\}}{(i-r)} \equiv \hat{f} > 0$$

for

$$w^u(2+r) > [w^s - (1+i)h] + \lambda(w^s - w^u)$$

Bequest Dynamics

$$b_{t+1} = \begin{cases} (1-\alpha)\{A_t w^u (2+r+\lambda) + (1+r) b_t\} & b_t \in [0, f] \\ (1-\alpha)\{A_t [w^s (1+\lambda) - (1+i) h] + (1+i) b_t\} & b_t \in [f, A_t h] \\ (1-\alpha)\{A_t [w^s (1+\lambda) - (1+r) h] + (1+r) b_t\} & b_t \in [A_t h, \infty] \end{cases}$$

Bequest Dynamics

Let $\hat{b}_{t+1} \equiv b_{t+1}A_{t+1}$

$$\hat{b}_{t+1} = \begin{cases} \left[\frac{1-\alpha}{1+\lambda}\right]\left\{w^u(2+r+\lambda)+(1+r)\hat{b}_t\right\} & \hat{b}_t \in [0,(\hat{f})] \\ \left[\frac{1-\alpha}{1+\lambda}\right]\left\{[w^s(1+\lambda)-(1+i)h]+(1+i)\hat{b}_t\right\} & \hat{b}_t \in [\hat{f},h] \\ \left[\frac{1-\alpha}{1+\lambda}\right]\left\{[w^s(1+\lambda)-(1+r)h]+(1+r)\hat{b}_t\right\} & \hat{b}_t \in [h,\infty] \end{cases}$$

$\Rightarrow$ The dynamical system is unaffected qualitatively by labor-augmenting technological progress

Sufficient Conditions for Multiple Steady-States

$$(1-\alpha)(1+r) < (1+\lambda)$$

$$(1-\alpha)(1+i) > (1+\lambda)$$

$$w^s(1+\lambda) - (1+i)h < 0$$

$\Rightarrow$ The system is characterized by multiple steady-state, where the unstable equilibrium

$$\hat{g} = \frac{(1-\alpha)[(1+i)h - w^s(1+\lambda)]}{[(1-\alpha)(i+i) - (1+\lambda)]} > 0$$

Income Per Capita in the Long Run

- Income of a skilled individual in the second period of life (wage and capital income)

$$I_2^s = w^s + (\bar{b}^s - h)r$$

- Income of an unskilled individual in the second period of life (wage and capital income)

$$I_2^u = w^u + (\bar{b}^u + w^u)r$$

- Income of an unskilled individual in the first period of life (only wage income)

$$I_1^u = w^u$$

Income Per Capita in the Long Run

- Aggregate income in the steady-state

$$\bar{Y} = I_2^s \bar{l}^s + I_2^u \bar{l}^u + I_1^u \bar{l}^u$$

- Aggregate income (note: $\bar{l}^s + \bar{l}^u = 1$)

$$\begin{aligned} Y &= [w^s - rh + r\bar{b}^s]\bar{l}^s + [w^u(2+r) + r\bar{b}^u](1 - \bar{l}^s) \\ &= w^u(2+r) + r\bar{b}^u + [(w^s - rh) - w^u(2+r) + (\bar{b}^s - \bar{b}^u)]\bar{l}^s \end{aligned}$$

- Income per capita

$$\bar{y} = \bar{Y}/2$$

Skill Composition and Income Per Capita in the Long Run

- An increase in the fraction of skilled workers increases income per capita in the steady-state

$$\frac{\partial \bar{y}}{\partial \bar{l}^s} = [(w^s - rh) - w^u(2 + r) + (\bar{b}^s - \bar{b}^u)]/2 > 0$$

since

$$w^s - (1 + r)h > w^u(2 + r)$$

$$\bar{b}^s > \bar{b}^u$$

- An increase in g reduces income per capita in the steady-state

$$\frac{\partial \bar{y}}{\partial g} = \frac{\partial \bar{y}}{\partial \bar{l}^s} \frac{\partial \bar{l}^s}{\partial g} < 0$$